M A T H E M A T I C S   T E A C H E R

Learning & Teaching

PK 12

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GPS: Using Games to Explore Composing and Decomposing

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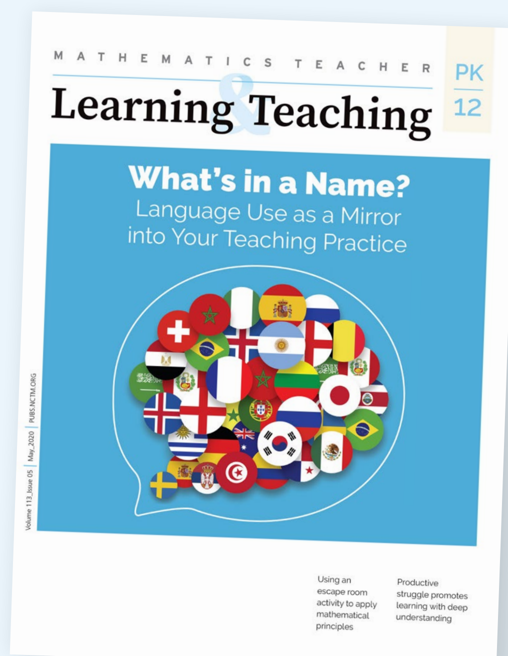

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

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GPS: Using Games to Explore Composing and Decomposing

Growing Problem Solvers provides four original, related, classroom-ready mathematical tasks, one for each grade band. Together, these tasks illustrate the trajectory of learners' growth as problem solvers across their years of school mathematics.

Nicole Anthony, Caitlin McPherson, Anne Molloy, Frances Vincent, and Katherine Ariemma Marin

In June's Growing Problem Solvers, we provide a series of games to help students explore composition and decomposition of numbers and algebraic expressions.

Students who interact with numbers flexibly and conceptually have strong number sense (Boaler, 2022). Students who possess strong number sense have an understanding of the structure of numbers and the ways in which numbers can be composed from or decomposed into units. Whole numbers are composed of units of 1s, and larger whole numbers of unitized groups of 10s and tens of 10s. Fractions are composed of unit fractions, with a fraction such as $\frac{3}{4}$ being composed of three $\frac{1}{4}$ units. Algebraic expressions can also be composed from and decomposed into smaller units. For example, $4x$ is composed of four units of x. Understanding these structures and the ways that numbers can be composed and decomposed is essential to students' development of strong number sense. When students understand the ways that numbers can be composed from and decomposed into smaller units, they can use that understanding to reason about different sizes and classifications of numbers (Van de Walle et al., 2023). This understanding of the structures of numbers is also an essential element of Standard of Mathematical Practice 7: "Look for and Make Use of Structure" (National Governors Association Center for Best Practices & Council of Chief State School Officers [NGA Center & CCSSO], 2010).

A great way for students to develop and strengthen their number sense and develop fluency with numbers is through games. Games provide enjoyable practice, opportunities for students to hear one another's thinking, and can serve as a formative assessment tool (Bay-Williams & Kling, 2019). In this month's Growing Problem Solvers, we provide students across the grade bands opportunities to build number sense with games. Each game is focused on composing and decomposing and building number sense by making sense of the ways that numbers and expressions can be put together and taken apart. These tasks address content standards across the grades. Those standards are listed in Table 1. These games also address Mathematical Practice 7: Look for and make use of structure across all grade bands (NGA Center & CCSSO, 2010).

Table 1 Standards Addressed in Tasks

Grade Band	Standard(s) Addressed
PK–2	**1.NBT.2** Understand that the two digits of a two-digit number represent amounts of tens and ones.
3–5	**4.NF.3.B** Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using drawings or visual fraction models.
6–8	**7.NS.A.** Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. 1. Apply and extend previous understandings of addition and subtraction to add and subtract integers and other rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make zero. For example, a hydrogen atom has zero charge because its two constituents are oppositely charged; If you open a new bank account with a deposit of $30 and then withdraw $30, you are left with a $0 balance.
9–12	**A.SSE.2** Use the structure of an expression to identify ways to rewrite it, for example, see x^4 − y^4 as (x^2)^2 − (y^2)^2, thus recognizing it as the difference of squares that can be factored as (x^2 − y^2)(x^2 + y^2). **A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.

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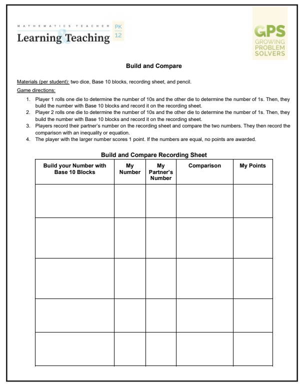
PK–2

BUILD AND COMPARE

In this game (link online), students will compose and compare two-digit numbers. Students will build models to represent their numbers using Base 10 blocks and write an inequality or equation to show how they compared the numbers. Students will compare their number with their partner's to determine who has the biggest number to win a point; another version is that they could get a point for the smallest sum. If the sums are equal, no points are awarded. This can be played with physical or virtual manipulatives; a good set of virtual manipulatives can be found at CoolMath4Kids (link online). While students play, ask them to identify patterns in the inequalities they create. Push students to share their strategies by asking, "How did you choose which die should represent the tens and which die should represent the ones?" Ask students to describe their strategy for making larger or smaller numbers.

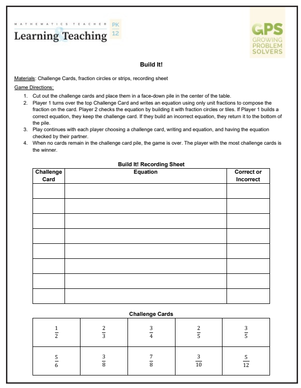


This game could be modified in a variety of ways to support a wide variety of students. For example, students could work to create the largest or smallest number by using two of the same-colored dice and selecting which die will represent the 10s and which will represent the 1s, or teachers could provide two different-colored dice and identify one color as the 10s and the other as the 1s. The game could be extended for students to work on adding two-digit numbers by adding the two players' numbers together and finding the sum after comparing them.

3–5

BUILD IT!

In this game (link online), students will compose fractions from unit fractions with like and unlike denominators. To play this game, students will need to understand what a unit fraction is (e.g., $\frac{1}{2}, \frac{1}{6}$) and how they can be used to compose a larger fraction. For example, $\frac{5}{6}$ is composed of $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$. But it could also be composed of a different set of unit fractions, such as $\frac{1}{3}+\frac{1}{3}+\frac{1}{6}$. Ask students to consider how they could build a fraction like $\frac{6}{8}$ using only unit fractions and share the different responses with the whole group. Be sure to record responses as equations. For example, $\frac{6}{8}=\frac{1}{2}+\frac{1}{4}$ and $\frac{6}{8}=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$. Have students build each response with fraction circles or tiles to confirm that the equation is true. This will model game play where one player writes an equation and the second player checks the accuracy of the equation with manipulatives.



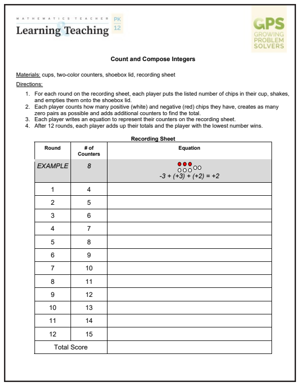
While they are playing, encourage students to use the manipulatives to explore how different unit fractions might work in an equation. One way to increase the challenge of the game is to require students to use a variety of unit fractions in their equations rather than repeating the same fraction multiple times. Ask, "How could you use fewer unit fractions in your equation?" Challenge students to include as many different unit fractions as possible in the same equation. As the game progresses, push students to compare multiple equations for the same challenge card and identify how the equations are the same and different.

6–8

COUNT AND COMPOSE INTEGERS

In this game (link online), students will compose integers from units of +1 and −1. The game focuses on the use of zero pairs. A pair of +1 and −1 are a zero pair because their combined total is equal to zero. Students will use two-color counters to play this game. Be sure to clearly identify which color will represent −1 and which color will represent +1. In most instances, red is used to represent −1 and white or yellow is used to represent +1.

As students play, ask them, “What do you notice about your total? How does it relate to the number of chips of each color?” Encourage students to describe their groups of chips using the terms *opposite* and *additive inverse*. As pairs finish playing, have students exchange recording sheets to check one another’s work. Ask them to compare their equations and push students to begin to generalize what they notice about using zero pairs when adding integers. Consider ending class with an exit ticket, asking students to write a generalization about using zero pairs when adding integers.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Count and Compose Integers

Materials: cups, two-color counters, shoebox lid, recording sheet

Directions:

1. For each round on the recording sheet, each player puts the listed number of chips in their cup, shakes, and empties them onto the shoebox lid.
2. Each player counts how many positive (white) and negative (red) chips they have, creates as many zero pairs as possible and adds additional counters to find the total.
3. Each player writes an equation to represent their counters on the recording sheet.
4. After 12 rounds, each player adds up their totals and the player with the lowest number wins.

Recording Sheet

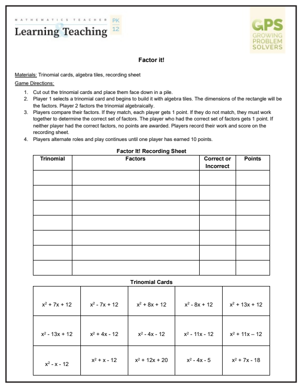
Round	# of Counters	Equation
EXAMPLE	*8*	*-3 + (+3) + (+2) = +2*
1	4	
2	5	
3	6	
4	7	
5	8	
6	9	
7	10	
8	11	
9	12	
10	13	
11	14	
12	15	
Total Score		

9–12

FACTOR IT!

In this game (link online), students will factor trinomials using both manipulatives and algebraic methods. The goal is for students to practice factoring and to provide opportunities for students to discover and understand the sign rules when factoring trinomials. In addition, students will have the opportunity to compare factoring methods and justify their reasoning with a partner. Students will use algebra tiles to represent trinomials. Those representations can be organized into a rectangle where the dimensions represent the binomial factors. If you do not have algebra tiles, you can use virtual manipulatives; a good set can be found on the Didax website (link online).

While students play, ask them, “What do you notice about the signs in the binomials as they relate to the signs in the original trinomials?” Ask students to look for patterns in the relationships between trinomials and their factors. After students have played, lead them in a whole group discussion about their noticings and wonderings about trinomials and their binomial factors. The purpose of the game and accompanying discussion is to discover the significance of the sign of the linear term as it relates to the sign of the constant term. In the summary discussion, be sure to focus on students’ conceptual understanding rather than the procedures used to factor algebraically.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Factor it!

Materials: Trinomial cards, algebra tiles, recording sheet

Game Directions:

1. Cut out the trinomial cards and place them face down in a pile.
2. Player 1 selects a trinomial card and begins to build it with algebra tiles. The dimensions of the rectangle will be the factors. Player 2 factors the trinomial algebraically.
3. Players compare their factors. If they match, each player gets 1 point. If they do not match, they must work together to determine the correct set of factors. The player who had the correct set of factors gets 1 point. If neither player had the correct factors, no points are awarded. Players record their work and score on the recording sheet.
4. Players alternate roles and play continues until one player has earned 10 points.

Factor It! Recording Sheet

Trinomial	Factors	Correct or Incorrect	Points

Trinomial Cards

$x^2 + 7x + 12$	$x^2 - 7x + 12$	$x^2 + 8x + 12$	$x^2 - 8x + 12$	$x^2 + 13x + 12$
$x^2 - 13x + 12$	$x^2 + 4x - 12$	$x^2 - 4x - 12$	$x^2 - 11x - 12$	$x^2 + 11x – 12$
$x^2 - x - 12$	$x^2 + x - 12$	$x^2 + 12x + 20$	$x^2 - 4x - 5$	$x^2 + 7x - 18$

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